# Hierarchy from Top Triple to Base-3 Triples

| Level | Name(s) | Internal triples | 3-base coordinates |
|---|---|---|---|
| **Top Triple** | **Gravity – Information – Cognition** | – | – |
| **Plane 1: Gravity** | | **Space – Time – Thought** | |
| | **Space** | – | Topology, Geometry, Scale |
| | **Time** | – | Proper-Duration ($\tau$), Causal-Order ($\chi$), Natural-Phase ($\varphi$) |
| | **Thought** | – | Utility, Processor, Memory |
| **Plane 2: Information** | | **Cyber – Logic – Entropy** | |
| | **Cyber** | – | Connections, Transport, Data |
| | **Logic** | – | Syntax, Semantics, Proof |
| | **Entropy** | – | Configurational, Informational, Intentional |
| **Plane 3: Cognition** | | **Context – Power – Meaning** | |
| | **Context** | – | Domain, Group, State |
| | **Power** | – | Change, Energy, Matter |
| | **Meaning** | – | Form, Narrative, Emotion |

*Summary:* 1 top-level triple → 3 planes → 9 mid-level triples → **33** base-level coordinates.

## 1 Hierarchical Structure

The model is organized in three levels:

- **Top Triple**: The highest-level concepts—Gravity, Information, and Cognition—serve as overarching domains.
- **Planes**: Each top-level concept defines a "plane" comprising three mid-level aspects, e.g., Gravity splits into Space, Time, and Thought.
- **Base-3 Coordinates**: Each mid-level aspect further decomposes into three fundamental variables (totaling 9 per plane and 33 overall).

## 2 Variables and Their Interpretation

Each base-3 coordinate is denoted by a single symbol, chosen to hint at its meaning:

- **Gravity Plane**: $T, G, Z$ (Space), $\tau, \chi, \phi$ (Time), $u, pr, m$ (Thought)
- **Information Plane**: $c, tr, d$ (Cyber), $\sigma, \mu, \pi$ (Logic), $E_c, E_i, E_{in}$ (Entropy)
- **Cognition Plane**: $D, Gr, St$ (Context), $\Delta, E, M$ (Power), $\phi, \eta, \psi$ (Meaning)

Each variable represents a state-dependent quantity, e.g., $c(x,t)$ for cyber connections evolving over space and time, or $u(t)$ for a thought-related scalar evolving only in time.

## 3 Equation Types and Notation

Four classes of differential systems are used:

1. **Continuum PDEs** (e.g., diffusion-reaction):

$$\frac{\partial T}{\partial t} = D_T \nabla^2 T + \kappa_T \nabla \cdot (C \nabla T) - \lambda_T E_c \tag{1}$$

2. **Mixed PDE/ODE** (for variables partly spatial, partly lumped):

$$\dot{\tau} = a_\tau \Omega - b_\tau E_c, \quad \frac{\partial \chi}{\partial t} = D_\chi \nabla^2 \chi + a_\chi \nabla \cdot (\Omega \nabla T) \tag{2}$$

3. **ODE / Stock-Flow Models** (logical or thought states):

$$\dot{\sigma} = \alpha_\sigma c - \beta_\sigma \sigma - \rho_\sigma B, \quad \dot{u} = \alpha_u pr - \beta_u u \tag{3}$$

4. **Delay/Integral ODEs and DAEs** (memory effects and conservation):

$$\dot{\psi} = \alpha_\psi \int_0^t \eta(s) e^{-\gamma(t-s)} ds - \beta_\psi \psi, \quad \dot{E} = \alpha_E Gr - \beta_E E \tag{4}$$

## 4 Putting It All Together: Example Systems

- **Gravity Plane Example** (Continuum PDE for $G$):

$$\frac{\partial G}{\partial t} = D_G \nabla^2 G + \kappa_G (T - G) - \lambda_G E_i \tag{5}$$

  Models how "gravity" interactions diffuse in space, are driven by differences with "space" $T$, and damped by informational entropy $E_i$.

- **Information Plane Example** (Network flow for $d$):

$$\dot{d} = -\lambda_d L_d + \alpha_d tr - \beta_d d \tag{6}$$

  Captures data flow on a sparse graph (with Laplacian $L_d$), sourced by transport $tr$ and lost via decay $\beta_d$.

- **Cognition Plane Example** (Compartment ODE for $Gr$):

$$\dot{Gr} = \alpha_{Gr} G - \beta_{Gr} Gr \tag{7}$$

  Links "meaningful change" to the context variable $G$ through a simple gain–loss dynamic.

# 5 Conclusion

This notation framework systematically arranges 33 interacting state variables into conceptually coherent groups, each governed by differential equations tailored to their physical or informational roles. By recognizing when to use PDEs, ODEs, DAEs, or integral forms—and by interpreting each symbol in its hierarchical context—students can build, analyze, and simulate rich multi-domain dynamical systems.

# Differential Systems for Base-3 Triples

## 1. Gravity Plane

| Triple | Variables | Recommended Form | Example Equations |
|---|---|---|---|
| **Space** | $T$, $G$, $Z$ | Continuum PDE | $\partial_t T = D_T \nabla^2 T + \kappa_T \nabla \cdot (C \nabla T) - \lambda_T E_c$<br>$\partial_t G = D_G \nabla^2 G + \kappa_G (T - G) - \lambda_G E_i$<br>$\partial_t Z = D_Z \nabla^2 Z + \kappa_Z \lvert G \rvert - \lambda_Z E_{in}$ |
| **Time** | $\tau$, $\chi$, $\varphi$ | Mixed PDE/ODE | $\partial_t \chi = D_\chi \nabla^2 \chi + a_\chi \nabla \cdot (\Omega \nabla T)$<br>$\dot{\tau} = a_\tau \Omega - b_\tau E_c$<br>$\dot{\varphi} = a_\varphi \Omega - b_\varphi E_i$ |
| **Thought** | $u$, $pr$, $m$ | ODE + integral | $\dot{u} = \alpha_u pr - \beta_u m$<br>$\dot{pr} = \alpha_{pr} u(1 - pr/P_{\max}) - \beta_{pr} pr$<br>$\dot{m} = \alpha_m u - \beta_m m + \int_0^t u(s) e^{-\gamma(t-s)} ds$ |

## 2. Information Plane

| Triple | Variables | Best Form | Example Equations |
|---|---|---|---|
| **Cyber** | $c$, $tr$, $d$ | Graph-PDE / Network Flow | $\dot{c} = -\lambda_c L c + s_c$<br>$\dot{tr} = -\lambda_{tr} L tr + \alpha_{tr} c - \beta_{tr} tr$<br>$\dot{d} = -\lambda_d L d + \alpha_d tr - \beta_d d$ |
| **Logic** | $\sigma$, $\mu$, $\pi$ | ODE/Discrete Update | $\dot{\sigma} = \alpha_\sigma c - \beta_\sigma \sigma - \rho_\sigma B$<br>$\dot{\mu} = \alpha_\mu \sigma - \beta_\mu \mu$<br>$\dot{\pi} = \alpha_\pi \mu - \beta_\pi \pi$ |
| **Entropy** | $E_c$, $E_i$, $E_{in}$ | Hybrid | $\partial_t E_c = D_{Ec} \nabla^2 E_c + \gamma_c \nabla \cdot (c \nabla T) - \epsilon_c E_c$<br>$\partial_t E_i = D_{Ei} \nabla^2 E_i + \gamma_i \nabla \cdot (tr \nabla d) - \epsilon_i E_i$<br>$\dot{E_{in}} = \gamma_{in} \lvert \nabla \eta \rvert - \epsilon_{in} E_{in}$ |

## 3. Cognition Plane

| Triple | Variables | Best Form | Example Equations |
|---|---|---|---|
| **Context** | $D$, $Gr$, $St$ | Compartment ODEs | $\dot{D} = \alpha_D T - \beta_D D$<br>$\dot{Gr} = \alpha_{Gr} G - \beta_{Gr} Gr$<br>$\dot{St} = \alpha_{St} Z - \beta_{St} St$ |
| **Power** | $\Delta$, $E$, $M$ | Balance ODE/DAE | $\dot{\Delta} = \alpha_\Delta D - \beta_\Delta \Delta$<br>$\dot{E} = \alpha_E Gr - \beta_E E$<br>$\dot{M} = \alpha_M St - \beta_M M$ |
| **Meaning** | $\varphi$, $\eta$, $\psi$ | Delay/Integral ODE | $\dot{\varphi} = \alpha_\varphi \Delta - \beta_\varphi \varphi$<br>$\dot{\eta} = \alpha_\eta \varphi(t - \tau_d) - \beta_\eta \eta$<br>$\dot{\psi} = \alpha_\psi \int_0^t \eta(s) e^{-\gamma(t-s)} ds - \beta_\psi \psi$ |

## Summary Table

| **Equation Type** | **Assigned Triples** | **Rationale** |
|---|---|---|
| Continuum PDE | Space, $\chi$, $E_c$, $E_i$ | Vary over space. |
| Graph-PDE | Cyber | Sparse networks. |
| ODE/Stock-Flow | Logic, Thought, Context, Power | Lumped states. |
| Delay/Integral ODE | Memory, Meaning | Memory/history dominate. |
| DAE | Power (Energy, Matter) | Conservation constraints. |