Hierarchy from Top Triple to Base-3 Triples

Natural Dimensions

The top-level dimensions, referred to as *Natural Dimensions*, consist of:

- **Gravity**
- **Information**
- **Cognition**

Plane Dimensions and Their Elemental Variables

Each Natural Dimension expands into three *Plane Dimensions*, each comprising three *Elemental Variables*:

Gravity Plane Dimensions

Space Variables: T (Topology), G (Geometry), Z (Scale)

Time Variables: τ (Proper-Duration), χ (Causal-Order), φ (Natural-Phase)

Thought Variables: u (Utility), pr (Processor), m (Memory)

Information Plane Dimensions

Cyber Variables: c (Connections), tr (Transport), d (Data)

Logic Variables: σ (Syntax), μ (Semantics), π (Proof)

Entropy Variables: E_c (Configurational), E_i (Informational), E_{in} (Intentional)

Cognition Plane Dimensions

Context Variables: D (Domain), Gr (Group), St (State)

Power Variables: Δ (Change), E (Energy), M (Matter)

Meaning Variables: ϕ (Form), η (Narrative), ψ (Emotion)

Artificial Dimension

An alternative *Artificial Dimension* reinterprets the planes by mixing elements across Natural Dimensions:

Gravity Plane $\rightarrow$ Cyber, Time, Thought

Information Plane $\rightarrow$ Space, Meaning, Entropy

Cognition Plane $\rightarrow$ Context, Logic, Meaning

Mirror Mapping

Each Natural variable X is paired with an Artificial mirror $X^\star$.

Natural	**Artificial Mirror**	**Notation**
Space Topology T	Space Topology $T^\star$	$(T, T^\star)$
Time Proper-Duration τ	Time $\tau^\star$	$(\tau, \tau^\star)$
Thought Utility u	Thought $u^\star$	$(u, u^\star)$
Cyber Connections c	Cyber $c^\star$	$(c, c^\star)$

(Complete the remaining 23 pairs similarly.)

Mixed-Reality Coordinate

Define a blended coordinate:

$$M_X = (1 - \alpha)X + \alpha X^\star,$$

where $\alpha \in [0, 1]$ interpolates between natural ($\alpha = 0$) and artificial ($\alpha = 1$) realms.

Coupled Dynamics

Each pair $(X, X^\star)$ evolves via:

$$\frac{\partial X}{\partial t} = F[X] - \lambda_{\text{mix}}(X - X^\star), \tag{1a}$$

$$\frac{\partial X^\star}{\partial t} = F^\star[X^\star] + \lambda_{\text{mix}}(X - X^\star). \tag{1b}$$

Example: Geometry vs. Geometry*

$$\frac{\partial G}{\partial t} = D_G \nabla^2 G + \kappa_G (T - G) - \lambda_G E_i - \lambda_{\text{mix}}(G - G^\star), \tag{2}$$

$$\frac{\partial G^\star}{\partial t} = D_G^\star \nabla^2 G^\star + \kappa_G^\star (T^\star - G^\star) - \lambda_G^\star E_i^\star + \lambda_{\text{mix}}(G - G^\star). \tag{3}$$

Example: Connections vs. Connections*

$$\dot{c} = -\lambda_c L_c c + s_c - \lambda_{\text{mix}}(c - c^\star), \tag{4}$$

$$\dot{c}^\star = -\lambda_c^\star L_c c^\star + s_c^\star + \lambda_{\text{mix}}(c - c^\star). \tag{5}$$

Observable Output

Interactions and visualizations access only the mixed-reality field M_X:

$$M_X(x, t) = (1 - \alpha)X(x, t) + \alpha X^\star(x, t).$$

Optionally, α may vary over time for dynamic immersion.

Implementation Steps

1. Duplicate each Natural equation to create its Artificial twin.
2. Add coupling terms $\pm\lambda_{\mathrm{mix}}(X - X^{\star})$ symmetrically.
3. Define mixed variables M_X for downstream modules.
4. Choose α and λ_{mix} to tune interaction strength.
5. Confirm system symmetry and conservation properties.

Extensions

- Formulate a block-matrix representation for all variable pairs.
- Introduce cross-variable coupling (off-diagonal mixing).
- Simulate benchmark cases to study blending effects.

Summary: 1 top-level triple $\rightarrow$ 3 planes $\rightarrow$ 9 mid-level triples $\rightarrow$ **33** base-level coordinates.

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1 Hierarchical Structure

The model is organized in three levels:

- **Top Triple**: The highest-level concepts—Gravity, Information, and Cognition—serve as overarching domains.
- **Planes**: Each top-level concept defines a "plane" comprising three mid-level aspects, e.g., Gravity splits into Space, Time, and Thought.
- **Base-3 Coordinates**: Each mid-level aspect further decomposes into three fundamental variables (totaling 9 per plane and 33 overall).

2 Variables and Their Interpretation

Each base-3 coordinate is denoted by a single symbol, chosen to hint at its meaning:

- **Gravity Plane**: T, G, Z (Space), τ, χ, ϕ (Time), u, pr, m (Thought)
- **Information Plane**: c, tr, d (Cyber), σ, μ, π (Logic), E_c, E_i, E_{in} (Entropy)
- **Cognition Plane**: D, Gr, St (Context), Δ, E, M (Power), ϕ, η, ψ (Meaning)

Each variable represents a state-dependent quantity, e.g., $c(x,t)$ for cyber connections evolving over space and time, or $u(t)$ for a thought-related scalar evolving only in time.

3 Equation Types and Notation

Four classes of differential systems are used:

1. **Continuum PDEs** (e.g., diffusion-reaction):

$$\frac{\partial T}{\partial t} = D_T \nabla^2 T + \kappa_T \nabla \cdot (C \nabla T) - \lambda_T E_c \tag{6}$$

2. **Mixed PDE/ODE** (for variables partly spatial, partly lumped):

$$\dot{\tau} = a_\tau \Omega - b_\tau E_c, \quad \frac{\partial \chi}{\partial t} = D_\chi \nabla^2 \chi + a_\chi \nabla \cdot (\Omega \nabla T) \tag{7}$$

3. **ODE / Stock-Flow Models** (logical or thought states):

$$\dot{\sigma} = \alpha_\sigma c - \beta_\sigma \sigma - \rho_\sigma B, \quad \dot{u} = \alpha_u pr - \beta_u u \tag{8}$$

4. **Delay/Integral ODEs and DAEs** (memory effects and conservation):

$$\dot{\psi} = \alpha_\psi \int_0^t \eta(s) e^{-\gamma(t-s)} ds - \beta_\psi \psi, \quad \dot{E} = \alpha_E Gr - \beta_E E \tag{9}$$

4 Putting It All Together: Example Systems

- **Gravity Plane Example** (Continuum PDE for G):

$$\frac{\partial G}{\partial t} = D_G \nabla^2 G + \kappa_G (T - G) - \lambda_G E_i \tag{10}$$

Models how "gravity" interactions diffuse in space, are driven by differences with "space" T, and damped by informational entropy E_i.

- **Information Plane Example** (Network flow for d):

$$\dot{d} = -\lambda_d L_d + \alpha_d tr - \beta_d d \tag{11}$$

Captures data flow on a sparse graph (with Laplacian L_d), sourced by transport tr and lost via decay β_d.

- **Cognition Plane Example** (Compartment ODE for Gr):

$$\dot{Gr} = \alpha_{Gr} G - \beta_{Gr} Gr \tag{12}$$

Links "meaningful change" to the context variable G through a simple gain–loss dynamic.

5 Conclusion

This notation framework systematically arranges 33 interacting state variables into conceptually coherent groups, each governed by differential equations tailored to their physical or informational roles. By recognizing when to use PDEs, ODEs, DAEs, or integral forms—and by interpreting each symbol in its hierarchical context—students can build, analyze, and simulate rich multi-domain dynamical systems.

Differential Systems for Base-3 Triples

1. Gravity Plane

Triple	**Variables**	**Recommended Form**	**Example Equations**
Space	T, G, Z	Continuum PDE	$\partial_t T = D_T \nabla^2 T + \kappa_T \nabla \cdot (C \nabla T) - \lambda_T E_c$ $\partial_t G = D_G \nabla^2 G + \kappa_G (T - G) - \lambda_G E_i$ $\partial_t Z = D_Z \nabla^2 Z + \kappa_Z \lvert G \rvert - \lambda_Z E_{in}$
Time	τ, χ, φ	Mixed PDE/ODE	$\partial_t \chi = D_\chi \nabla^2 \chi + a_\chi \nabla \cdot (\Omega \nabla T)$ $\dot{\tau} = a_\tau \Omega - b_\tau E_c$ $\dot{\varphi} = a_\varphi \Omega - b_\varphi E_i$
Thought	u, pr, m	ODE + integral	$\dot{u} = \alpha_u pr - \beta_u m$ $\dot{pr} = \alpha_{pr} u(1 - pr/P_{\max}) - \beta_{pr} pr$ $\dot{m} = \alpha_m u - \beta_m m + \int_0^t u(s) e^{-\gamma(t-s)} ds$

2. Information Plane

Triple	**Variables**	**Best Form**	**Example Equations**
Cyber	c, tr, d	Graph-PDE / Network Flow	$\dot{c} = -\lambda_c L c + s_c$ $\dot{tr} = -\lambda_{tr} L tr + \alpha_{tr} c - \beta_{tr} tr$ $\dot{d} = -\lambda_d L d + \alpha_d tr - \beta_d d$

Logic	σ, μ, π	ODE/Discrete Update	$\dot{\sigma} = \alpha_\sigma c - \beta_\sigma \sigma - \rho_\sigma B$ $\dot{\mu} = \alpha_\mu \sigma - \beta_\mu \mu$ $\dot{\pi} = \alpha_\pi \mu - \beta_\pi \pi$
Entropy	E_c, E_i, E_{in}	Hybrid	$\partial_t E_c = D_{Ec} \nabla^2 E_c + \gamma_c \nabla \cdot (c \nabla T) - \epsilon_c E_c$ $\partial_t E_i = D_{Ei} \nabla^2 E_i + \gamma_i \nabla \cdot (tr \nabla d) - \epsilon_i E_i$ $\dot{E_{in}} = \gamma_{in} \lvert \nabla \eta \rvert - \epsilon_{in} E_{in}$

3. Cognition Plane

Triple	**Variables**	**Best Form**	**Example Equations**
Context	D, Gr, St	Compartment ODEs	$\dot{D} = \alpha_D T - \beta_D D$ $\dot{Gr} = \alpha_{Gr} G - \beta_{Gr} Gr$ $\dot{St} = \alpha_{St} Z - \beta_{St} St$
Power	Δ, E, M	Balance ODE/DAE	$\dot{\Delta} = \alpha_\Delta D - \beta_\Delta \Delta$ $\dot{E} = \alpha_E Gr - \beta_E E$ $\dot{M} = \alpha_M St - \beta_M M$
Meaning	φ, η, ψ	Delay/Integral ODE	$\dot{\varphi} = \alpha_\varphi \Delta - \beta_\varphi \varphi$ $\dot{\eta} = \alpha_\eta \varphi(t - \tau_d) - \beta_\eta \eta$ $\dot{\psi} = \alpha_\psi \int_0^t \eta(s) e^{-\gamma(t-s)} ds - \beta_\psi \psi$

Summary Table

Equation Type	**Assigned Triples**	**Rationale**
Continuum PDE	Space, χ, E_c, E_i	Vary over space.
Graph-PDE	Cyber	Sparse networks.
ODE/Stock-Flow	Logic, Thought, Context, Power	Lumped states.
Delay/Integral ODE	Memory, Meaning	Memory/history dominate.
DAE	Power (Energy, Matter)	Conservation constraints.